ONE-DIMENSIONAL DIFFUSIONAL MODEL OF HETEROGENEOUS (GAS – FILM) DETONATION

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The detonation process in a tube filled with a gaseous oxidant (oxygen) and which has a "thick" layer of fuel (carbon with a low vapor pressure) deposited over its entire perimeter is examined; the weight ratio of fuel to oxidant considerably exceeds the stoichiometric ratio. It is assumed that the rate of heat release is determined by the diffusional (noninstantaneous) process of mixing of the vaporizing fuel and the oxidant. An estimate is made of the effect on the detonation parameters of heterogeneity in the composition over the cross section of the tube and of friction and heat losses. Dependences of the detonation parameters (propagation velocity, pressure profile in the front, distance to the Chapman–Jouquet plane) on the thermophysical properties of the fuel and oxidant are obtained.

1. It has been established experimentally [1, 2] that the propagation of a detonation process is possible in a tube filled with a gaseous oxidant and on the wall of which is deposited a film of fuel (with a low vapor pressure under the initial conditions). The front of a heterogeneous detonation consists of a shock wave and an extended (not less than two to four tube diameters) combustion zone behind it. A detonation is possible with almost infinitely high ratios of the weight of the fuel on the walls of the tube to the weight of the gaseous oxidant in the volume.

In [3-5], devoted to theoretical consideration of the process, the instantaneous mixing of the fuel entering the volume of the tube with the oxidant was assumed. In [3] the rate of heat release was determined by the kinetics of the chemical reaction of oxidation, and it was concluded that the propagation of the process is impossible without a sufficiently intense supply of fuel. In [4, 5] it was assumed that the fuel entering the volume of the tube from the walls burns instantly, and the Chapman–Jouquet plane was identified with the plane of total vaporization of the fuel film. The velocity defect in the propagation of the process in comparison with the velocity of homogeneous detonation of the same composition was determined by the heat losses and losses to friction. The authors of [4, 5] confined themselves to an examination of processes for "thin" fuel layers, i.e., for ratios of the weight of fuel on the walls of the tube to the weight of gaseous oxidant in the volume close to the stoichiomettic ratio.

A schematic picture of the detonation front obtained on the basis of experimental data [5-7] is shown in Fig. 1. Here 0 is the shock front, L_1 is the distance from the shock front to the joining of the boundary layers, and L_2 is the distance from the shock front to the plane of completion of heat release.

The following simplifying assumptions are made in the derivation of the equations:

(1) the gas density ρ, pressure p, temperature T, and stream velocity relative to the shock front u are assumed to be constant over a cross section of the tube.

(2) The specific gas constant R and specific heat capacity c_p of the gas are assumed to be constant over a cross section of the tube.

(3) The ratio of heat capacities γ is assumed to be constant over a cross section of the tube, independent of the distance to the shock front, and equal to the value of γ at the Chapman–Jouquet plane for homogeneous gas detonation of a stoichiometric mixture of fuel with oxidant [8]

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$$\gamma = 1.17 \tag{1.1}$$

(4) The rate of the chemical reaction is assumed to be infinitely high; the heat release is determined by the processes of turbulent diffusion of the fuel vaporizing into the volume of the tube; the fuel concentration is assumed to be variable over a cross section of the tube.

(5) The heat release H_0 per gram of oxidant due to the chemical reaction is assumed to be independent of the composition of the products and to equal the corresponding value for homogeneous detonation of a mixture of stoichiometric composition [9]

$$H_o = D_H^2 / 2\,(\gamma^2 - 1)\, c_o' \tag{1.2}$$

where D_H and c_o' are the detonation velocity and weight concentration of oxidant in a mixture of stoichiometric composition.

(6) The value of the blowing parameter B [described below by Eqs. (2.1) and (2.2)] is assumed to be independent of the distance to the shock front.

In the coordinate system connected with the shock front the stationary equations of continuity, momentum, and energy with allowance for friction, heat losses, mass supply, and heat release due to the chemical reaction have the form

$$\begin{gathered}
S\rho u = S\rho_1 D + l\int_0^x m\,dx \\
S\,(\rho u^2 + p) = S\,(\rho_1 D^2 + p_1) + Dl\int_0^x m\,dx + l\int_0^x \tau_w\,dx \\
S\rho u\left(\frac{u^2}{2} + c_p T\right) = S\rho_1 D\left(\frac{D^2}{2} + c_p{}^1 T_1\right) + \left(\frac{D^2}{2} + c_p T_w\right) l\int_0^x m\,dx + S\rho_1 D H_o W + Dl\int_0^x \tau_w\,dx - l\int_0^x q_w\,dx
\end{gathered} \tag{1.3}$$

where D is the velocity of propagation of the shock front, S and l are the cross-sectional area and perimeter of the tube, x is the distance from the shock front, m is the rate of mass supply of fuel from a unit of surface, τ_w is the shearing stress of friction, q_w is the heat flux to a unit of surface of the fuel film, W is the fraction of oxygen which has undergone the reaction, and the index 1 pertains to the initial state of the oxidant.

The equation of state and the expression for the square of the speed of sound have the form

$$p = \rho RT, \qquad a^2 = \gamma RT \tag{1.4}$$

Let us change to dimensionless coordinates and to the parameters

$$\begin{gathered}
\pi = p / p_1, \ \sigma = \rho_1 / \rho, \ M = u / a, \ M_1 = D / a_1 \\
\eta_1 = \int_0^x \frac{lm}{S\rho_1 D}\,dx, \quad \eta_2 = \int_0^x \frac{l\tau_w}{S\rho_1 D^2}\,dx, \quad \eta_3 = \int_0^x \frac{lq_w}{S\rho_1 D^3}\,dx
\end{gathered} \tag{1.5}$$

where π is the dimensionless pressure, σ is the dimensionless density, M is the Mach number of the stream relative to the shock front, M_1 is the Mach number of the shock front, η_1 is the dimensionless blowing of the flue — the ratio of the mass flow of the vaporizing fuel to the flow of oxidant at the cross section x of the tube, η_2 is the dimensionless friction, and η_3 is the dimensionless heat losses. By eliminating π and σ from the equations of motion in dimensionless form we obtain an equation of fourth power relative to M:

$$\frac{\gamma^2 M^2 \left[M^2/2 + 1/(\gamma - 1)\right]}{(\gamma M^2 + 1)^2} = \frac{1}{2} + \frac{1 - Z}{2\,(\gamma^2 - 1)} \tag{1.6}$$

$$Z = 1 - 2\,(\gamma^2 - 1)\left\{(1 + \eta_1)\left[\left(\frac{D_H}{D}\right)^2 \frac{W}{2\,(\gamma^2 - 1)\,c_o'} + \frac{1}{2}(1 + \eta_1) + (\eta_2 - \eta_3)\right][1 + \eta_1 + \eta_2]^{-2} - \frac{1}{2}\right\} \tag{1.7}$$

In the expression for Z we neglect values on the order of $(M_1)^{-2}$ in comparison with unity.

By subtracting $^1/_2$ from both sides of (1.6) we obtain a quadratic equation relative to $(\gamma M^2 + 1)$. Hence,

$$M_\pm = \left(\frac{\gamma + 1}{\gamma}\,\frac{1}{1 \pm \sqrt{Z}} - \frac{1}{\gamma}\right)^{1/2} \tag{1.8}$$

The plus and minus signs correspond to the two branches of the solution of Eq. (1.6).

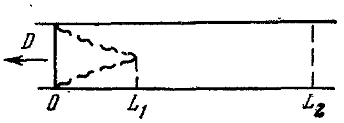

Fig. 1

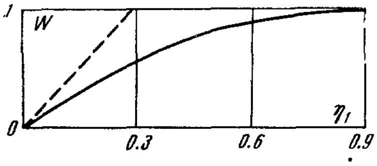

Fig. 2

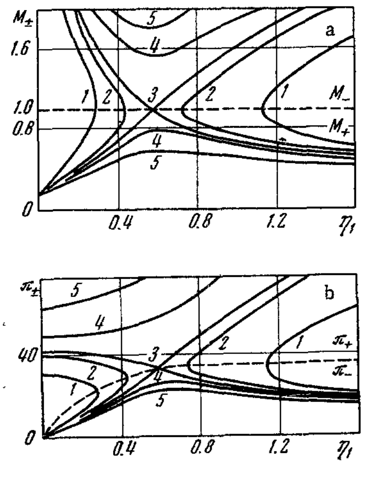

Fig. 3

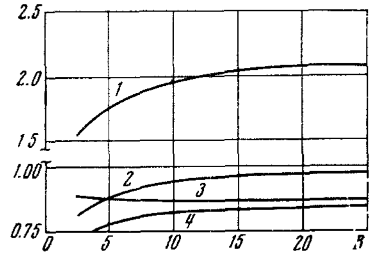

Fig. 4

For π and σ we obtain

$$\pi_\pm = \frac{\gamma_1 M_1^2}{\gamma M_\pm^2 + 1}(1+\eta_1+\eta_2), \quad \sigma_\pm = \frac{\gamma M_\pm^2}{\gamma M_\pm^2 + 1}\,\frac{(1+\eta_1+\eta_2)}{(1+\eta_1)^2} \qquad (1.9)$$

where γ_1 is the ratio of heat capacities of the oxidant in the initial state. In the absence of blowing ($\eta_1 = 0$), of friction ($\eta_2 = 0$), and of heat losses ($\eta_3 = 0$) Eq. (1.9) is converted into the corresponding equation for homogeneous detonation.

2. For an analysis of the flow it is necessary either to know the concrete dependence on x of the functions η_1, η_2, η_3, and W or to express all these functions through one of them. Since in the present work we are confined to an examination of the detonation process for "thick" layers of fuel, it is advisable to take the dimensionless blowing η_1 as the parameter and to express the other values through it.

To describe the process of mass supply behind the front of the traveling shock wave we will use propositions developed in the theory of the boundary layer with blowing. Following [10], we write the expression for the rate of mass supply in the form

$$m = B\rho(D - u)\,\mathrm{St} \qquad (2.1)$$

Here B is the blowing parameter – the basic characteristic for the description of the boundary layer at a permeable surface – and St is the Stanton number. Assuming that the heat supplied to the surface of the fuel film is expended on the heating of the fuel and its subsequent vaporization, we obtain, following [10], an expression for the blowing parameter for the case of liquid films

$$B = \frac{H_e - H_w}{c_L(T_w - T_1) + r(T_w)} \approx \frac{H_0}{c_L(T_2 - T_1)} \qquad (2.2)$$

where $H_e - H_w$ is the difference in total specific enthalpies at the axis and at the wall of the tube, and c_L, $r(T_w)$, and T_2 are the specific heat capacity, heat of vaporization, and critical temperature of the fuel. The first expression is an exact determination of the blowing parameter B; the expression for the difference in total enthalpies $H_e - H_w$ incorporates the difference in thermal enthalpies, a kinetic term allowing for viscous dissipation, and the heat of combustion per gram of oxidant. In the approximate expression for B, which we will use hereafter, only the heat release due to the chemical reaction is allowed for. Since for liquid unsaturated hydrocarbons in a wide range of T_w (starting with the boiling temperature at one atmosphere and higher) the value $c_L(T_w - T_1) + r(T_w)$ varies by no more than 20%, one can assume with good accruacy than $T_w = T_2$ in (2.2).

For "frozen" fuel films one must allow, in addition for the heat of melting and for warming the solid phase to the melting temperature in the expression for B (2.2). Since in real systems in which the propagation of a heterogeneous detonation is possible the heat of combustion is considerably greater than the heat of vaporization, we have

$$B \gg 1 \qquad (2.3)$$

With allowance for (2.1) the expression for η_1 is converted to the form

$$\eta_1 = B\frac{l}{S}\int_0^x \sigma^{-1}\left(1 - \frac{u}{D}\right)\mathrm{St}\,dx \qquad (2.4)$$

We can write the expression for the heat flux to the surface of the fuel film in the form

$$q_w = \rho\,(D - u)\,\mathrm{St}\,(H_e - H_w) \approx \rho\,(D - u)\,\mathrm{St}\,H_0 \qquad (2.5)$$

and for the heat losses in dimensionless form we obtain

$$\eta_3 = \frac{H_o}{D^2}\,\frac{l}{S}\int_0^x \sigma^{-1}\left(1-\frac{u}{D}\right)\mathrm{St}\,dx = \frac{H_o}{D^2}\eta_1 B^{-1} \tag{2.6}$$

The expression for the shearing stress τ_W with allowance for the Reynolds analogy $c_f/2 = \mathrm{St}$, where c_f is the local coefficient of friction, takes the form

$$\tau_w = \mathrm{St}\,\rho\,(D-u)^2 \tag{2.7}$$

The expression for the dimensionless friction has the form

$$\eta_2 = \frac{l}{S}\int_0^x \sigma^{-1}\left(1-\frac{u}{D}\right)\mathrm{St}\left(1-\frac{u}{D}\right)dx = \left\langle 1-\frac{u}{D}\right\rangle \eta_1 B^{-1} \tag{2.8}$$

where $<1-u/D>$ is the mean value of $1-(u/D)$ over the section 0x.

As seen from Eqs. (2.3), (2.6), and (2.8), η_2 and η_3 are considerably less than η_1. Since in Eq. (1.7) for Z the values η_1, η_2, and η_3 enter in the form of linear combinations, rough estimates are sufficient for the calculation of η_2 and η_3. In particular, for a powerful shock wave one can take $1-(u/D)$ directly for $<1-(u/D)>$:

$$\langle 1 - u/D\rangle = 2/(\gamma+1) = 0.92 \tag{2.9}$$

Let us find the relation between the amount of fuel vaporized and the amount of heat released. On the assumption of an infinite chemical reaction rate and of the constancy of ρ and u over a cross section of the tube, the heat release over the section 0x will be determined by the distribution of weight concentrations of oxidant c_O and fuel $c_E = 1 - c_O$ at the cross section x

$$H_oW = \frac{1}{S}\int_{(S)} H\,\frac{\rho u}{\rho_1 D}\,dS = (1+\eta_1)\,\frac{1}{S}\int_{(S)} H\,dS$$

$$H = \begin{cases} H_o c_o, & \text{if} \quad c_o \leqslant c_o' \\ H_E c_E, & \text{if} \quad c_o \geqslant c_o' \end{cases} \tag{2.10}$$

Here H, H_O, and H_E are the heat release per gram of mixture, per gram of oxidant, and per gram of fuel, respectively; H_O and H_E are related by the equation

$$H_o c_o' = H_E\,(1-c_o') \tag{2.11}$$

The choice of H in this form takes into account the fact that near the surface of the fuel the heat release due to the chemical reaction is determined by the oxygen concentration (here the fuel is in excess) whereas near the axis of the tube the heat release is determined by the fuel concentration.

The value η_1 is also determined by the distribution of fuel concentrations in the cross section x:

$$\frac{\eta_1}{1+\eta_1} = \frac{1}{S}\int_{(S)} c_E\,dS \tag{2.12}$$

Thus, the connection between H_OW and η_1 is determined by the concentration profile. The distribution of concentrations over the tube cross section will be assumed to be analogous to the distribution of concentrations in a turbulent boundary layer with blowing and combustion [11]:

$$c_o / c_o^e = [(y/\delta)^{1/n} + B/2\ \ (y/\delta)^{2/n}]\ [1 + B/2]^{-1} \tag{2.13}$$

Here c_O^e is the concentration of oxidant at the tube axis, y is the distance from the surface of the tube, and δ is the thickness of the diffusional boundary layer. In the section before the joining of the boundary layers ($\delta < d/2$, where d is the tube diameter) the oxidant concentration at the tube axis is constant ($c_O^e = 1$). After the joining of the boundary layers c_O^e decreases in proportion to the distance from the shock front.

The dependences obtained among η_2, η_3, H_OW, and η_1 permit the construction of dependences of $M_\pm$ and $\pi_\pm$ in the form of functions of the dimensionless blowing η_1. The following are taken as the parameters: the ratio D/D_H of the propagation velocity of the shock front to the propagation velocity of a homogeneous detonation of a stoichiometric mixture of the given fuel with the oxidant, the blowing parameter B, and the exponent 1/n in Eq. (2.13) for the distribution of concentrations.

As shown in [12], the value 1/n corresponds to the exponent in the velocity distribution for tubulent flow in the absence of blowing. If one neglects the effect on the shock front of disturbances developing at

TABLE 1

Substance studied	B	D_j, m/sec (calc.)	D_j, m/sec (expt.)	d, cm	Source
VM-4 vacuum oil	9.4	1897	1500	2.6	The authors
Axle grease	9.6	1899	1600	—	[16]
Frozen hexadecane	9.8	1901	1820 1950 1860	0.475 1.11 2.22	[15]
Hexadecane	10.5	1908	1000 1710 1860	0.475 1.11 2.22	[15]
Decane	12.8	1925	1000 1710 1860	0.475 1.11 2.22	[15]

the surface of the liquid fuel film [13], then n depends weakly on the Reynolds number $Re=(D-u)x/\nu$, which makes it possible to use rough estimates of Re in determining n. Taking as $(D-u)$ and the kinematic viscosity ν the corresponding values immediately behind the shock front we obtain $Re=2\cdot10^6\,xp_1$, where x is measured in cm and p_1 in atm. For normal initial pressure in tubes with a diameter d=1-10 cm the characteristic Reynolds number at x=d is $Re=2\cdot10^6$-$2\cdot10^7$, which corresponds to n=7 [14]. Since most experimental results are obtained under these conditions, the calculations presented below are conducted for n=7.

A dependence for the fraction W of reacted oxygen in proportion to the supply of fuel for the case of B=10 and n=7 is presented in Fig. 2 (solid curve). As seen from Fig. 2, by the time all the oxidant has reacted (W=1), the amount of fuel supplied considerably exceeds that necessary for stoichiometry: η_1 (W=1)=0.84, whereas for a stoichiometric composition $\eta_1=(1-c_0')/c_0'=0.29$. For the case of instantaneous mixing of fuel and oxidant the fraction of reacted oxidant is proportional to the amount of fuel supplied (dashed straight line in Fig. 2).

The behavior of the curves of $M_\pm$ and $\pi_\pm$ for the values B=10 and n=7 and different D/D_H are presented in Fig. 3 (curves 1, 2, 3, 4, and 5 correspond to D/DH=0.7, 0.8, 0.8203, 0.9, and 1.0). Immediately behind the shock front the flow must be subsonic and is described by the branch with the plus sign. At comparatively low values of D (curve 1) the flow has a discontinuous nature: the value of Z is negative in the section $0.28<\eta_1<1.13$. Physically, this means that the heat release in the section $0.28<\eta_1<1.13$ leads to the formation of shock waves propagating toward the shock front and increasing its intensity. Such a process cannot be stationary.

At comparatively large values of D (curve 4) the curve of M_+ is not discontinuous but it does not reach a value equal to unity at any point. The profile of pressure π_+ corresponding to this value of D/D_H is shown in Fig. 3b. From the dependences for M_+ and π_+ we find that the flow is related to the conditions in the initial section of the tube, and it can be stationary only at a certain compression (by a piston, for example) of the combustion products.

At a single value $D=D_j$ for the given B and n (curve 3) the dependence of M_+ is continuous and at the point $\eta_1=\eta_1^*=0.56$ takes on a value equal to unity. The contact of the M_+ and M_- curves, as well as of the π_+ and π_- curves, occurs at this point. The flow in some neighborhood behind the plane M=1 can be described either by the curve of M_+ or of M_-. Since in real processes a wave of rarefaction follows behind the front of the detonation wave, the pressure profile in some neighborhood behind the plane M=1 will be described by the curve of π_- and the flow by the curve of M_-.

For the given B=10 and n=7 at a propagation velocity of the shock front corresponding to $D/D_H=D_j/D_H=0.8203$ the flow has a continuous nature and proves to be independent of the conditions in the initial section of the tube: the disturbances from this region, weak waves of rarefaction, do not pass through the section η_1^* in which the Mach number of the stream relative to the shock front is equal to unity. The propagation of the process is determined by the heat release, mass supply, friction, and heat losses in the section $0\eta_1^*$. Such a process will be stationary and the term "Chapman–Jouquet detonation" is fully applicable to it.

Flow which is also described by the curve of M_+ beyond the plane M=1 could be realized in principle, but such a process would require a certain compression of the detonation products: the pressure must vary along the curve of π_+.

Both the relative velocity D_j/D_H of the Chapman–Jouquet detonation and the value η_1^* which characterizes the composition of the products in the Chapman–Jouquet plane (the plane $M=1$) can be determined similarly for other values of B and n.

3. The dependence $D_j(B)/D_H$ for $n=7$ is presented in Fig. 4 (curve 4). The corresponding dependence is also plotted there for $\varphi^* = \eta_1^* c_0'(1-c_0')$ (curve 1), where φ is the coefficient of fuel excess which characterizes the overenrichment of the composition in the Chapman–Jouquet plane.

The results of the calculations showed the slight dependence of the relative detonation velocity D_j/D_H and the composition φ^* in the Chapman–Jouquet plane on a blowing parameter $B > 7$. This circumstance permits the use of the approximate expression (2.2) for the calculation of B: the inaccuracy in the determination of B has little effect on the detonation parameters.

For detonation in tubes filled with oxygen at an initial temperature $T_1 = 293°K$ (20°C) and on the walls of which is deposited a film of liquid hydrocarbon fuel the blowing parameter [calculated from (2.2)] lies in the range of $8 < B < 17$, where all the assumptions and simplifications made above relative to B are valid.

The composition of products in the Chapman–Jouquet plane is considerably overenriched in comparison with the stoichiometric composition ($\varphi^* \approx 2$, $\eta_1 \approx 0.58$), although, as follows from Fig. 2, only 93% of the oxygen reacts in this case.

The overenrichment of the composition is a consequence of the "noninstantaneous" mixing of the fuel and oxidant. Since the rate of heat release is determined by the process of turbulent diffusion of fuel vapors, an excess amount of fuel must enter the layer near the wall as the oxygen at the axis of the tube is consumed.

In order to compare the decrease in the velocity of the heterogeneous detonation due to the overenrichment of the composition in the Chapman–Jouquet plane with the decrease in velocity due to friction and heat losses it is necessary to determine the connection between D_j/D_H and the values η_1, η_2, η_3, and W in the Chapman–Jouquet plane. As follows from (1.8) and (1.7), the following equation is valid in the Chapman–Jouquet plane:

$$\frac{D_j}{D_H} = \left\{\left[\frac{lW}{(1+\eta_1)c_0'}\right]^{1/2}\right\}\left\{\left[1+\gamma^2\left(\left(1+\frac{\eta_2}{1+\eta_1}\right)^2-1\right)+2(\gamma^2-1)\frac{\eta_3-\eta_2}{1+\eta_1}\right]^{-1/2}\right\} \qquad (3.1)$$

Let us designate the factor standing in the first curly brackets as θ_1^* and the factor in the second curly brackets as θ_2^*. Here θ_1^* is determined by the composition in the Chapman–Jouquet plane, while θ_2^* characterizes the effect of friction and heat losses on D_j/D_H. The dependences $\theta_1^*(B)$ and $\theta_2^*(B)$ for $n=7$ are presented in Fig. 4 (curves 3 and 2).

The overenrichment of the composition is the determining factor in the drop in the velocity of heterogeneous detonation compared with the velocity of homogeneous detonation of a mixture of stoichiometric composition. The velocity defect due to the overenrichment of the composition, characterized by the value $1-\theta_1^*$, reaches ~14%.

The contribution of friction and heat losses to the drop in the velocity of heterogeneous detonation, the value $1-\theta_2^*$, is relatively small and does not exceed 6%.

4. The comparison of the results of the one-dimensional diffusional model proposed in the present report with the results of experiments was conducted for three paramters: a) for the detonation velocities; b) for the characteristic distances to the joining of the boundary layers and to the completion of heat release; c) for the pressure profiles in the front of the detonation wave.

The calculated and experimental values of the propagation velocities of detonation in round tubes for several heterogeneous systems as well as the values of the blowing parameter B for these systems, are presented in Table 1. The velocity of homogeneous detonation $D_H = 2320$ m/sec was assumed to be the same for all the hydrocarbons indicated in the table and equal to the velocity of homogeneous detonation in a stoichiometric mixture of diethylcyclohexane ($C_{10}H_{20}$) and oxygen [8]. In the experiments $B = 9.4$–12.8. The calculated velocities of detonation in this range of B were $D_j = 1897$–1925 m/sec.

The results obtained agree well with the experimental velocities [15] for frozen hexadecane and for liquid decane and hexadecane for $d = 2.22$ cm.

In the experiments conducted with liquid decane and hexadecane in a tube 1.11 cm in diameter the velocities differ somewhat from the calculated velocities, while for a tube 0.475 cm in diameter they differ strongly. At the same time the results of experiments conducted with frozen hexadecane in these same tubes agree well with the calculation. The disagreement between the experimental and calculated values can be explained by the strong effect of disturbances in the surface of the liquid fuel film on the parameters of the stream at such small diameters.

Some of the disagreement between the calculated and experimental velocities obtained by the authors for viscous fuel (VM-4 vacuum oil) can be explained by the unevenness or "roughness" of the film surface produced when it was deposited on the wall of the tube.

For the comparison of the theoretical and experimental data relative to the characteristic distances L_1 and L_2, it is necessary to relate the "generalized coordinate" η_1 with the current coordinate x. From the integral equation (2.4) we obtain

$$x / d = 2.6 \cdot 10^2 \; [I / (B\Psi)]^{5/4} p_1^{1/4} d^{1/4} \tag{4.1}$$

$$I = \int_0^{\eta_1} \frac{d\eta_1}{\sigma^{-1} - (1 + \eta_1)}, \qquad \Psi^{-1} = 1 + 0.19 (\mu_o / \mu_E)^{1/3} B$$

Here Ψ is the decrease in the Stanton number due to the blowing of the fuel [17], μ_E and μ_o are the molecular weights of the blown in gas (fuel) and the oxidant, d is measured in cm, and p_1 is measured in atm. In the derivation of (4.1) the Stanton number was taken in the form $St = St_0 \Psi$, where $St_0 = 3.7 \cdot 10^{-2}\ Re^{-0.2}$ is the Stanton number behind the traveling shock wave in the absence of blowing [18].

For tubes with diameter d = 1-4 cm and normal initial pressure of the oxidant the length of the section up to the joining of the boundary layers is $L_1 = (2.5\text{-}3.5)d$. The joining of the boundary layers on the shadow photographs presented in [6, 7] takes place at distances $L_1 = (2\text{-}4)d$ from the shock front and is in satisfactory agreement with the calculation.

Since during diffusional combustion the flame front is located in a region of concentrations close to stoichiometric, i.e., in the interior of the boundary layers, the completion of heat release must take place at considerably greater distances from the shock front. The calculations give the following estimates for the distances from the shock front to the plane in which all the oxygen has reacted: $L_2 = (16\text{-}23)d$.

Since the heat release due to the chemical reaction makes the most important contribution to the heat flux to the surface of the tube, its completion leads to a decrease in the heat flux. The experimental values of the distance starting with which the heat flux to the surface of the tube decreases are $L_2 = (13\text{-}15)d$ according to [5] and agree well with the theoretical calculations.

There are no known experimental data on the determination of the coordinates of the Chapman–Jouquet plane. For the conditions indicated above the calculated value is $L^* = (5.3\text{-}7.5)d$.

It follows from a comparison of L_1, L_2, and L^* that the Chapman–Jouquet plane is located closer to the plane in which the joining of the boundary layers occurs (recorded on the shadow photographs) than to the plane of completion of heat release.

There is an absence of data in the literature on the measurement of the pressure in the front of a heterogeneous detonation wave propagating in a tube of circular cross section. Data are presented in [5] on the measurement of pressure in the front of a detonation wave propagating in a tube of square cross section with a fuel film deposited over the entire perimeter. In these experiments the propagation velocity of the detonation was 1700 m/sec, and the pressure exceeded the initial (atmospheric) pressure by 20 times and remained almost constant up to the plane of completion of heat release.

A calculated pressure profile for a Chapman–Jouquet detonation is presented in Fig. 3b (curve 3). The dimensionless pressure immediately behind the shock front is $\pi = 43$. The drop in pressure up to the Chapman–Jouquet plane does not exceed 20% of the pressure immediately behind the front.

Since the increase in pressure behind the front is proportional to the square of the propagation velocity of the detonation, a quantitative disparity is observed between the experimental and calculated profiles. However, the qualitative behavior of the curves – a very small drop in pressure with greater distance from the shock front – is about the same.

The satisfactory agreement of the experimental and calculated data allows one to conclude that the one-dimensional diffusional model correctly describes the process of propagation of a detonation in round tubes filled with oxygen and with a thick film of hydrocarbon fuel deposited on the walls.

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